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BEING CLEAR ABOUT THE EXPLANATION: A MATHEMATICAL EXAMPLE IN ARISTOTLE, *METAPHYSICA Θ*.9, 1051A26–9[1]

In *Metaphysica Θ*.9, 1051a21–33, Aristotle makes the point that geometrical proofs are discovered through auxiliary constructions, which only need to be actualized in order to make us see how the proof is going to work. He gives two examples, of which the first, his stock example of a triangle which is proved to have internal angles equal to two right angles, is clear enough. The second example, however, at 1051a26–9, has caused difficulties, of a twofold nature: first, it is not immediately clear what the mathematical argument presupposed by the example is; and second, the text is disputed, both in terms of what words to read and how to punctuate. In this note I want to argue that, though recent discussions of the example have more or less sorted out the mathematical issue, there still is a significant residual difficulty, and that this difficulty hangs together with the textual problems the example has given rise to.

I

According to the most recent editor, Jaeger,[2] Aristotle sketches the example as follows:

διὰ τί ἐν ἡμικυκλίῳ ὀρθὴ καθόλου; διότι ἐὰν ἴσαι τρεῖς, ἥ τε βάσις δύο καὶ ἡ ἐκ μέσου ἐπισταθεῖσα ὀρθή, ἰδόντι δῆλον τῷ ἐκεῖνο εἰδότι. (1051a26–9)

Aristotle is referring to the theorem that the angle in a triangle opposite to its base, with that base coinciding with the diameter of a semicircle and the apex of the triangle on the circumference, is always a right angle. But on what ground (*διὰ τί*) is this the case, or at least, what is the auxiliary construction and the argument Aristotle envisages in the present passage?

Over the years this issue has been discussed by quite a few scholars,[3] but I take it as established that Aristotle must be thinking of the diagram shown in Figure 1 opposite. For only thus have we just the three equal lines referred to, two, MA and MB, forming together the base of the triangle and the semicircle (*ἥ τε βάσις*

[1] I should like to thank István Bodnár and an anonymous referee for helpful comments.
[2] W. Jaeger (ed.), *Aristotelis* Metaphysica (Oxford, 1957).
[3] Most recently by S. Makin, *Aristotle*, Metaphysics *Book Θ* (Oxford, 2006), 13 and 234–7, and most convincingly and intelligently by H. Mendell, 'Two geometrical examples from Aristotle's *Metaphysics*', *CQ* 34 (1984), 359–72, from 363 onwards (in fact, an earlier draft of this paper is the unacknowledged ultimate source for every interpretation with the correct mathematical reconstruction of the proof). For others I refer to their discussions.

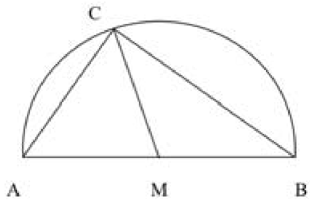

FIGURE 1

δύο) and one, MC, erected from the centre of the semicircle (καὶ ἡ ἐκ μέσου ἐπισταθεῖσα), which are equally long because they are all radii of the semicircle.[4]

The proof accompanying this diagram is based on two points. The first is that MA, MB and MC, all being equally long, form the legs of two isosceles triangles, whence it follows that in each of these two triangles the two angles at the third side are equal (by Euclid, *Elements* 1.5). The second is the theorem of the first example (1051a24–6), that the internal angles of a triangle are together equal to two right angles (*Elements* 1.32). This theorem is the only possible reference of ἐκεῖνο in τῷ ἐκεῖνο εἰδότι.[5] Moreover, Aristotle alludes elsewhere to that theorem in connection with the proof about the angle in a semicircle.[6] This gives us the following proof:

(1)	$\angle$ MAC + $\angle$ MBC + $\angle$ ACB = 2 right angles	*El.* 1.32
(2)	$\angle$ MAC = $\angle$ MCA	*El.* 1.5
(3)	$\angle$ MBC = $\angle$ MCB	*El.* 1.5
(4)	$\angle$ ACB = $\angle$ MCA + $\angle$ MCB	(diagram)
(5)	$\angle$ ACB = ½ ($\angle$ MAC + $\angle$ MBC + $\angle$ ACB)	(2)–(4)
(6)	$\angle$ ACB = ½ (2 right angles) = 1 right angle	(1), (5)

However, there is one serious problem with this reconstruction, and that is that in the second sentence of the passage quoted, at 1051a28, Aristotle seems to be saying of the line erected from the centre M that it is ὀρθή. Now in Greek mathematics a line which is ὀρθή is one which stands at right angles to another line. If that is to be the meaning of ὀρθή in the present passage, it will be impossible to give a plausible reconstruction of the mathematical proof Aristotle is referring to, as the proof would then not be general any more, but only concern the case that angle CMA is a right angle.[7]

[4] The fact that no other auxiliary lines are referred to excludes the proof of Euclid, *Elements* 3.31 as a possible reconstruction, for that depends crucially on the extension of BC beyond C.

[5] It cannot have a 'less determinate' reference, as Makin (n. 3), 237 would seem to allow for.

[6] See *An. Post.*1.1, 71a19–21: 'That every triangle has angles equal to two right ones, one knew beforehand, but that *this* thing in the semicircle is a triangle, one learned at the same time as inducing it.' And after that the discussion continues to be couched in terms of the application of the general theorem about triangles to this particular figure (71a26–9).

[7] Indeed, it is in order to accommodate this meaning of ὀρθή that T. Heath, *The Thirteen Books of Euclid's* Elements. *Translated from the Text of Heiberg with Introduction and Commentary* II *Books III–IX* (Oxford, 1925[2]), 63, and W.D. Ross, *Aristotle's* Metaphysics. *A Revised Text with Introduction and Commentary* (Oxford, 1924) 2.270–1, adopt an alternative reconstruction which, however, does not immediately prove generally that the angle in a semicircle is right; for the best criticism, see Mendell (n. 3), 363–5; cf. M. Burnyeat et al. (edd.), *Notes on Books Eta and Theta of Aristotle's* Metaphysics (Oxford, 1984), 148.

Makin, following Owen, suggests a solution to this problem by proposing that a line may also be called ὀρθή if it is just a straight line. This will not do, however. Though ὀρθός can be used to indicate the mere straightness of something, for example a road, and is perhaps on one occasion[8] used by Plato to indicate the straightness of a rectilinear figure,[9] it is never used thus in the case of a line in Greek mathematics nor, indeed, with anything else in the whole *corpus Aristotelicum*.[10] Moreover, Aristotle would be guilty of misleading language if he were to use ὀρθή suddenly and in such a deviant way for the mere straightness of a line in a context where the same word is used time and again for the rightness of an angle. Finally, in similar cases Aristotle does not explicate either that the non-circular lines are straight – customarily a mathematical line is a straight line in Aristotle, unless there is a contrast with non-straight lines to be drawn. It would be surprising if Aristotle were to deviate from this custom.

Thus the problem still stands: if we want to make mathematical sense of Aristotle's words, we must be able to understand ὀρθή within the context of the proof as sketched above, but it is completely unclear how that is possible.

II

Assuming this problem about the meaning of ὀρθή at 1051a28 to be really insoluble, most scholars have taken recourse to emending the text. Above I quoted the text as edited by Jaeger, but for various reasons it has been disputed, both as to what words to read and how to punctuate. Let me give the text as read by the majority of the manuscripts, starting from 1051a25 (the last sentence of the previous example):

εἰ οὖν ἀνῆκτο ἡ παρὰ τὴν πλευράν, ἰδόντι ἂν ἦν εὐθὺς δῆλον. διὰ τί ἐν ἡμικυκλίῳ ὀρθὴ καθόλου; διὰ τί ἐὰν ἴσαι τρεῖς, ἥ τε βάσις δύο καὶ ἡ ἐκ μέσου ἐπισταθεῖσα ὀρθή, ἰδόντι δῆλον τῷ ἐκεῖνο εἰδότι.

Apart from some minor variations, this is the text provided by all three old manuscripts cited consistently by Jaeger and Ross (J, E, A^b), but also by the three other, more recent, manuscripts[11] appearing in the stemma made by Harlfinger.[12]

[8] C. Mugler, *Dictionnaire historique de la terminologie géométrique des Grecs* (Paris, 1958), 314 claims that there are other passages in Plato, but none of them is mathematical.

[9] If we translate *Ti.* 53c5–8 as follows: 'Depth is, by all necessity, contained by a surface (τὴν ἐπίπεδον … φύσιν), and the rectilinear kind of surface figure (ἡ δ' ὀρθὴ τῆς ἐπιπέδου βάσεως) is composed of triangles.' However, I think it cannot be excluded that ὀρθή does not refer to the straightness of the edges, but rather to the straightness or evenness of the surface itself.

[10] The only possible exception I could find is at *Hist. an.* 4.2, 526b12, where Aristotle says that the lobster has on each of a number of body parts πρὸς τὰ ἔξω ἄκανθαν βραχεῖαν καὶ ὀρθήν, but there the idea of the thorn being erect, rather than just straight, seems to be much more important.

[11] Namely E^s (Scorialensis Y III 18 [13th c.]), I^b (Parisinus Coislinianus 161 [14th c.]) and M (Mediolanensis Ambrosianus 363 [F 113 sup.] [14th c.]) – as appears from the apparatus in G. Vuillemin-Diem (ed.), *Aristoteles Latinus* XXV 3.2 Metaphysica *Lib. I–XIV Recensio et Translatio Guillelmi de Moerbeka. Editio Textus* [hereafter referred to simply as *Editio Textus*] (Leiden, 1995).

[12] See D. Harlfinger, 'Zur Überlieferungsgeschichte der *Metaphysik*', in P. Aubenque (ed.), *Études sur la* Métaphysique *d'Aristote. Actes du VIe Symposium aristotelicum* (Paris, 1979), 7–36, at 27.

However, none of the modern editions read the text thus. Most editors[13] choose to follow some later manuscripts and the medieval translations in reading *διότι* rather than *διὰ τί* after *καθόλου*.[14] In itself, however, this does not solve the problem with *ὀρθή* – for that further emendation would be required. The only one proposed is by Christ, who in a note suggests reading a raised period after *ὀρθή*, so that we may translate: 'Because, when … [the angle is] right – [that] is clear to …'[15] Ross, on the other hand, does retain the words of the manuscripts, but punctuates differently, preferring the punctuation as suggested by Cannan: … *εὐθὺς δῆλον διὰ τί. ἐν ἡμικυκλίῳ ὀρθὴ καθόλου διὰ τί; ἐὰν ἴσαι κτλ.*, though not ruling out the divisions as read by pseudo-Alexander: *δῆλον. διὰ τί ἐν ἡμικυκλίῳ ὀρθή; καθόλου διὰ τί; ἐὰν ἴσαι κτλ.* (*In* Metaphysica 596.21–4).[16] Again, in itself such a repunctuation is not enough to solve the problem with *ὀρθή*. Therefore Mendell suggests as further possibilities, in combination with Cannan's punctuation, the insertion of *διὰ τί* or *ὅτι* before *ὀρθή*;[17] Cannan himself makes a suggestion which achieves the same effect, by changing *ἰδόντι δῆλον* into *διὰ τί; δῆλον*.[18] The most radical solution, ultimately preferred by Mendell, is to strike *ὀρθή* altogether.[19]

Apart from the problem with *ὀρθή*, which, as we have seen, is not immediately dealt with by most scholars, these emendations and repunctuations are meant to solve two problematic features the manuscript text is perceived to have. The first appears from Jaeger's claim, made in his apparatus, that *διὰ τί* cannot be combined with *δῆλον*, but must indicate the beginning of a question to which a clause starting with *ὅτι* or *διότι* provides the answer. Secondly, and more importantly, it seems to be a common presupposition in the debate that the long conditional clause in the reading of all these manuscripts *διὰ τι ἐὰν ἴσαι τρεῖς ἥ τε βάσις δύο καὶ ἡ ἐκ μέσου ἐπισταθεῖσα ὀρθή, ἰδόντι δῆλον* is odd and to be avoided. Rather than discussing all these proposals for emendation and repunctuation extensively,[20] I shall argue that these two features are not really problematic – in fact, once we see why not, it will be immediately obvious that the problem with *ὀρθή* has been solved as well.

As to the first feature alleged to be problematic, if Jaeger's claim is to be interpreted as implying that a sub-clause with *διὰ τί* cannot depend on *δῆλον*,

[13] In addition to Jaeger (see above), they include I. Bekker (ed.), *Aristotelis opera* II (Berlin, 1831), H. Bonitz, *Aristotelis* Metaphysica. *Commentarius* (Bonn, 1849) and W. Christ (ed.), *Aristotelis* Metaphysica (Leipzig, 1895); cf. Makin (n. 3), 13 and 273, and the revised Oxford translation in J. Barnes (ed.), *The Complete Works of Aristotle* (Princeton, 1984), 1660.

[14] Namely T (Vaticanus graecus 256 [1320]) and E^b (Venetus Marcianus 211 [13th/14th c.]) – see the apparatus in G. Vuillemin-Diem (ed.), *Aristoteles Latinus* XXV 2 Metaphysica *Lib. I–X, XII–XIV. Translatio Anonyma sive 'Media'* (Leiden, 1976) and in id., *Editio Textus*. The oldest medieval translation containing *Metaphysica Θ*, the 'media' (from the end of the twelfth century), as well as that produced by Willelm van Moerbeke (1260–70) read as if the Greek had *διότι* (*eo quod* and *quia* respectively). (As it is, though, Willelm van Moerbeke based his translation on the earlier one and on manuscript J, but opted for *quia* rather than *propter quid*, as he should have done if he had followed J – thus his translation does not have any independent textual authority.) Finally, Jaeger in his apparatus claims that pseudo-Alexander reads *διότι* (though admitting that he also considers *διὰ τί*). However, I cannot find anything in pseudo-Alexander remotely indicative of *διότι*.

[15] Christ (n. 13), ad loc.

[16] See Ross (n. 7), 270–1.

[17] Mendell (n. 3), 368–9.

[18] See Ross (n. 7), ad loc.

[19] Mendell (n. 3), 367 and 369.

[20] Again one can find the best discussion in Mendell (n. 3).

then it is simply false, for there are quite a few places in Aristotle where we do have such a sub-clause.[21] If, on the other hand, Jaeger's claim is based on his understanding of the point of the present passage, namely that what should be clear is the explanation, to be introduced with ὅτι or διότι, and not the διὰ τί question asking for that explanation, then I fail to see the relevance of that distinction in the context. For if it is clear διὰ τί, then it is also clear ὅτι or διότι. Moreover, in the context it should not be a mere fact which is clear, but that this fact is explanatory. This appears from the first example adduced by Aristotle:

On what ground is the triangle two right angles? Because the angles around a single point are equal to two right angles. If, then, the line parallel to the side had been drawn upwards (ἀνῆκτο),[22] it would have been clear on seeing it. (1051a24–6)

What would have been clear on seeing it? Not merely *that* the angles around a single point are equal to two right angles (seeing that does not require any diagram), but that it is *because* of that, that the triangle is two right angles.

As to the second perceived problem, this disappears just by placing two additional commas, one after διὰ τί and one before ὀρθή:

διὰ τί, ἐὰν ἴσαι τρεῖς, ἥ τε βάσις δύο καὶ ἡ ἐκ μέσου ἐπισταθεῖσα, ὀρθή, ἰδόντι δῆλον τῷ ἐκεῖνο εἰδότι.

Though the structure of the sentence is thus still rather cumbersome, containing a very long protasis between the only two words of the apodosis, Aristotle follows a very common grammatical pattern occurring in sentences featuring a διὰ τί question. For example, in the pseudo-Aristotelian *Mechanics* we read:

On what ground (διὰ τί), if (ἐὰν) one places a heavy axe on a piece of wood and puts a heavy weight on top of it, does it not cleave (οὐ διαιρεῖ) the wood to any considerable extent?[23] (19, 853b14–15)

Archimedes phrases his purely mathematical research questions similarly in the introduction to *On Conoids and Spheroids*:

On what ground (διὰ τί), if (εἴ κα) a segment of the right-angled conoid be cut off by a plane at right angles to the axis, will the segment so cut off be half as large again as the cone which has the same base as the segment and the same axis?[24]

In particular, the pseudo-Aristotelian *Problems* are littered with similar examples:

[21] *An. Post.* 1.31, 88a15, 2.14, 98a11 and 18 (cf. 2.2, 90a27), *Eth. Eud.* 2.1, 1219b17, *Metaph.* E.2, 1026b26, *Rh.* 2.9, 1387b4, and *Sens.* 6, 445b21.

[22] For a helpful discussion of the meaning of ἀνῆκτο here, see Mendell (n. 3), 362.

[23] Translation taken from Barnes (n. 13), 1309, with marginal adaptations; other examples with ἐάν occur at 850a3, 853a32 and 857a22, and with ὅταν at 851b6 and 857b9.

[24] See J. Heiberg and E.S. Stamatis (edd.), *Archimedis Opera Omnia cum Commentariis Eutocii* (Stuttgart, 1972), 248.11–15; translation taken from T.L. Heath (tr.), *The Works of Archimedes* (Cambridge, 1912^{2}), 100. Cf. also *De Conoidibus et Sphaeroidibus* 248.15, 250.24, 252.3, 254.22 and 256.10.

On what ground (διὰ τί), if (ἐὰν) the winter is characterized by south winds and rainy and if the spring is dry with the wind in the north, are both the spring and the summer unhealthy?[25] (1.9, 860a12)

Such διὰ τί questions are characteristic for what one may call 'scientific problems': questions asking for the explanation of a certain phenomenon.[26] Most of these questions specify in the ἐὰν clause (if they have one) circumstances which are part of the fact to be explained or in which the fact to be explained comes about, as in the examples quoted from the *Mechanics* and Archimedes. That is, however, not the case in the example cited from the *Problems*. For there the ἐὰν clause rather mentions facts which actually seem to make it more difficult to come up with a good explanation.[27] From there it is not so far to an ἐὰν clause specifying circumstances which make it easier to see the right explanation, as an auxiliary construction in a diagram is meant to do. In fact, one could see the present διὰ τί question, including the auxiliary construction, as a reformulation of the original problem διὰ τί ἐν ἡμικυκλίῳ ὀρθὴ καθόλου; This reformulation would then constitute a step in answering the original problem.

Moreover, on the basis of these parallel passages we may now translate the sentence as follows:

On what ground, if there are three equal lines, the base [consisting of] two and the one erected from the centre, [the angle in the semicircle is] right, is clear on seeing it for anyone who knows that [*sc.* the theorem about the angles in a triangle].

Thus the problem with ὀρθή has also disappeared, as it can have its normal meaning, referring here to the rightness of the angle ACB in the triangle – the reading of the manuscripts allows for a perfectly smooth interpretation, both in mathematical and philological terms: as soon as the auxiliary construction has been realized, anyone who knows the earlier theorem about the angles in a triangle will be clear about the explanation for the angle in a semicircle being right.[28]

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[25] Translation taken from Barnes (n. 13), 1321, with marginal adaptations; in total I count 26 examples of διὰ τί, ἐὰν …, … in the *Problemata*, but there are also examples with ὅταν (11) and εἰ (1).

[26] More often than not Aristotle seems to refer to such questions when mentioning προβλήματα in scientific contexts (rather than to dialectical problems, which are of the form: πότερον … ἢ οὔ; see *Top.* 1.4, 101b28–34). This is particularly clear in e.g. *An. Post.* 2.15. Alexander of Aphrodisias, *In Topica* 63.11–12 (V. Rose, *Aristotelis Fragmenta* [Stuttgart, 1967] fr. 112) mentions a work *Περὶ προβλημάτων* in which Aristotle apparently drew the distinction between dialectical problems and these scientific problems.

[27] Cf. also *Pr.* 7.2, 886a29–31, and, with εἰ, 19.9, 918a29–30.

[28] If we follow Aristotle's account of the same proof in *An. Post.* 2.11, 94a28–36, the explanation we are supposed to see immediately is that the angle in a semicircle is half two right angles – which is a combination of seeing that the angle in a semicircle is half the sum of the internal angles of a triangle (proposition [5] in the proof above) and knowing that this sum is two right angles. As the knowledge about this sum is referred to separately, we may conclude that what the auxiliary construction by itself makes us see is this proposition (5).